A Distributed Coordination Framework for On-line Scheduling and Power Demand Balancing of Households Communities

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Abstract— **The development of energy management systems (EMS) has attracted increasing attention during the last years. One of the main goals of EMS is to balance the power usage and generation, while also maintaining the quality of life (QoL) of the users. In this paper, a distributed coordination framework for on-line scheduling of appliances and control of the aggregated power demand of households is proposed. Each household consists of a set of appliances, and each appliance is modeled using a probabilistic generative model of its power usage profile, which can be updated through the day to indicate changes in user preferences. The coordination framework is formulated as a receding horizon distributed optimization, where the households' QoL, and the deviation from a scheduled usage are taken into account. The implemented distributed optimization can be seen as a negotiation among the households and a coordinator: the coordinator seeks to balance the aggregated power consumption by minimizing the deviation from a scheduled aggregated power usage, while each household tries not to deviate much from its preferred usage pattern.**

I. INTRODUCTION

Power grid management systems are evolving towards a Internet-like network of distributed devices that produce / consume energy and that can exchange information about their energy use. This is allowing to control these networked devices in a coordinated way, and thus a more effective management of the available resources. The increasing need for energy efficiency, together with environmental issues, and the deployment of advanced metering infrastructure is driving the design of these energy management systems (EMS). Within this context, we are interested in the development of a demand-side management system where: *i*) the households in a community coordinate their power usage, and *ii*) control is not enforced on the users but the schedule is coordinated taking into account their Quality of Life (QoL).

Several demand-side management systems exist [1], [2], [3], going from approaches where the utility applies direct load control or uses of dynamic pricing to drive power usage [4], [5], [6], to recently proposed methods that seek to be consumer-friendly [7], [8], [9], [10], [11] by taking QoL into account. The scheduling of electric loads that can be deferred, such as the charging of electric vehicles (EV) [12], [13], together with the control of thermostatically controlled loads [14], [8], such as air conditioning, are some of the enablers and drivers of the development of demand management systems. Continuing the line of research proposed in [11], we use a distributed coordination method for balancing the aggregated power of a community of households[1] that takes into account the QoL of each household.

Coordinated and distributed power balancing is growing in importance because power balancing is directly related to the cost and stability of the power grid. In particular, the steadily increase of devices such as EV is requiring and enabling a coordinated scheduling [15], [13], [12]. This can help ensure stability by reducing peaks and valleys, thus addressing the supply-demand balance by smoothing the demand and responding to frequency variations. The stability issue has been traditionally managed by ancillary services (in a reactive way), thought it can be addressed more effectively when the demand-side contributes to it (see e.g. [16], [17]). The distributed control of power resources, which has been studied in the utility side [18], [19], [17] and in the demand side [15], can reduce the need to pass information about local cost functions [20] usually determined by a large amounts of (private) data. In our context this private data consists of daily appliance power usage patterns.

We propose the on-line distributed coordination of power usage within a community of households, with the main goal of balancing the power usage profile but also taking into account QoL and privacy. Our scenario is close to [21], which tries to minimize the deviation from a day-ahead power usage schedule and also takes into account the households' QoL. In order to maintain the QoL and to effectively schedule the appliances, modeling the flexibility of appliance usage is of key importance. While the modeling of flexibility has been discussed in the literature (e.g., [5], [22], [23], [10]), we utilize a generative probabilistic model, which can be learned from data of daily usage patterns, to describe the QoL through the flexibility of appliances. This model can describe controllable and non controllable appliances and can be updated through the day (similarly to [24]), thus taking into account load uncertainty and usage information.

Our work builds up on the ideas proposed in recent work [11], where a coordinated scheduling for balancing the power consumptions in households by exploiting a distributed algorithm called the alternating direction method of multipliers (ADMM) [25] and a probabilistic mode-switching model (which will be described in Section II) was developed. In particular, ADMM is now attracting great attention of the control community (e.g., [26], [20], [27], [28], [29]) for its fast and robust convergence in real problems. In particular,

This work was supported by CREST from the Japan Science and Technology.

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[1] A household is understood as any group of appliances that can be controlled (not only at homes), such as offices, buildings, etc.

it has been used in distributed electric vehicle charging control [30] [31] and optimal power flow [32] [33] [25]. The main contribution of this paper is to extend the scheme introduced in [11] to an on-line scheduling problem and to show in simulation how effective the proposed ADMM-based receding horizon optimization is for distributed power-balancing problems. In particular, we show how the proposed method responds to changes in the households' behavior, modeled as changes in the probabilistic generative model.

The paper is structured as follows. We first give an overview of the framework (Section II), and afterwards we continue with a detailed formulation (Section III), where we describe the appliance and household models, the coordinator role, the receding horizon formulation, the distributed optimization and negotiation, and the optimal mode scheduling of the appliances. Later we present simulation results (Section IV) to finally conclude (Section V).

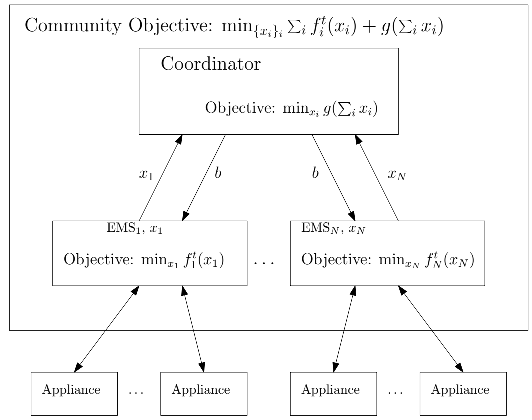

Fig. 1. System diagram. At each time step t, an iterative negotiation takes places where the coordinator seeks to balance the aggregated power usage profile, as measured by $\sum_i x_i$, and each household seeks to minimize the loss in QoL associated to its power usage profile x_i, as measured by $f_i^t(x_i)$.

II. Framework

The purpose of the proposed framework (see Fig. 1) is to implement a system to coordinate the power usage of a community of households, that takes into account the QoL. The coordination of the households is done by the community itself, instead of being controlled directly from the utility company. The community has a coordinator, which together with the households seek to balance the aggregated power usage of the community through the day by following a day-ahead coordinated plan. In the same way, each household has an EMS that manages the devices at the household. (In the following we will use "household" to refer to the household's EMS when clear from context). It is assumed that the households have some degree of flexibility in their appliance usage, e.g. by controlling the power usage level or scheduling some appliances.

The coordinator provides support, having two main goals: to negotiate, with all households, a day-ahead schedule of the community's aggregated power usage, and to perform an on-line negotiation for adjusting to deviations from the plan that may occur through the day[2].

The proposed framework makes a few assumptions, all of which have been already applied in real-world scenarios: *i*) the household can measure the power usage of its appliances, *ii*) the household can control or schedule some of the appliances, and *iii*) in the case when the appliance can not be controlled, the appliance (or the user) may inform the household its most likely schedule and power usage.

III. Formulation

A. Appliances and Household

Each household $i = 1, \dots, N$ consists of a set of N_i appliances, $a_i = 1, \dots, N_i$, where appliance a_i has a power consumption profile $x_{i,a_i} \in \mathbb{R}^T$, with T the number of time-slots within a time period (e.g. a day). For example the power usage profile for one day can be represented with $T = 144$ time-slots of 10 minutes. In the same way, each household i, has an aggregated power consumption profile which corresponds to the sum of the power usage of the individual appliances: $x_i = \sum_{a_i} x_{i,a_i}$, with $x_i = (x_i(1), \dots, x_i(T))$, and $x_i(t)$ the aggregated power usage of household i at time t, with $1 \le t \le T$.

We consider that appliance a_i can evaluate how difficult is to realize a power profile x_{i,a_i} through a function $f_{i,a_i} : \mathbb{R}^T \to \mathbb{R}$, and that the appliance a_i can be scheduled and controlled –with a given degree of uncertainty– through its associated control signal $u_{i,a_i} \in \mathcal{U}_{i,a_i}$. A control signal could indicate when to start using the appliance (e.g. in the case of a washing machine), when to start charging the appliance (e.g. an electric vehicle), or set the power usage level of the appliance (e.g. the intensity of the lighting or the air conditioner). This control should be selected such that it maximizes the QoL of the household i but taking into account the uncertainties in achieving the profile x_{i,a_i}:

$$f_{i,a_i}(x_{i,a_i}) = \min_{u_{i,a_i} \in \mathcal{U}_{i,a_i}} \left[f^u_{i,a_i}(u_{i,a_i}) + f^{x|u}_{i,a_i}(x_{i,a_i}, u_{i,a_i}) \right]. \tag{1}$$

The first term, $f^u_{i,a_i}(u_{i,a_i})$, represents the dissatisfaction (degree of deviation from common pattern usage) associated with control u_{i,a_i}, and it can be understood as a measure of the flexibility in the appliance usage, or as a measure of the uncertainty of the control u_{i,a_i} being applied. The second term, $f^{x|u}_{i,a_i}(x_{i,a_i}, u_{i,a_i})$, represents the uncertainty of achieving the power usage profile x_{i,a_i} for the control signal u_{i,a_i}, thus when a profile x_{i,a_i} is not achievable by a given control signal u_{i,a_i}, the function $f^{x|u}_{i,a_i}(x_{i,a_i}, u_{i,a_i})$ would give a large (possible infinite) value, thus also allowing to encode restrictions or uncertainties in achieving certain profiles. In Section III-F we present an implementation of such function $f_{i,a_i}(x_{i,a_i})$ using a generative probabilistic model. We assume that the household i has access to $f_{i,a_i}(x_{i,a_i})$, and that it can schedule actions u_{i,a_i} for the appliances.

The overall household's goal is to minimize the aggregated

[2]We assume that the community buys energy from the utility and that the energy cost is related to the flatness of the power usage of the community, however we do not analyze how this cost may be split among the households and instead we focus on the coordinated scheduling of the appliances.

dissatisfaction and the uncertainty associated with the power usage profile of all appliances:

$$\min_{\{x_{i,a_i}\}_{a_i}} \left[\sum_{a_i=1}^{N_i} f_{i,a_i}(x_{i,a_i}) \right]. \tag{2}$$

It is important to recall that the control of the appliances will not be enforced: it is applied only when possible. For example, the charging of an electric vehicle can start only once the vehicle has been plugged, or the specific timing could be specified by the user. Thus for each time step t, there will be a possible different cost function for appliance a_i. We assume that the appliance (or its user) may inform the household's EMS, through the day, the usage timing of the appliance (e.g. when the rice cooker will be on), thus the household will have an updated $f^t_{i,a_i}(x_{i,a_i})$ the moment this information is given. In general we will consider that the household may update the model at each time step $f^t_{i,a_i}(x_{i,a_i})$, but we will omit the t index unless necessary. Given that the cost function may change over time, and that the control may not be applied as planned, at each time step the households and the coordinator will negotiate a new power usage profile for the remainder of the day. This is expected to happen through the day, thus the control signals will be updated and applied at each time step t.

B. Coordinator

The objective of the community is to balance the aggregated power usage. This will be measured by a shared a global cost $g(\sum_{i=1}^{N} x_i)$, with $x_i = \sum_{a_i=1}^{N_i} x_{i,a_i}$, that depends on the aggregated power usage profile $\sum_{i=1}^{N} x_i$ (the power usage profile of the community). The coordinator's goal is to help find a good trade-off between the shared cost $g(\sum_{i=1}^{N} x_i)$, and the QoL of all households. This multi-objective optimization problem will be solved through the following single-objective optimization formulation:

$$\min_{\{x_{i,a_i}\}_{i,a_i}} \left[\sum_{i=1}^{N} \sum_{a_i=1}^{N_i} f_{i,a_i}(x_{i,a_i}) + g\left(\sum_{i=1}^{N} \sum_{a_i=1}^{N_i} x_{i,a_i} \right) \right]. \tag{3}$$

This last problem will be solved by the coordinator and the households by using a receding horizon formulation, through a distributed optimization, and will be implemented as an iterative negotiation between the coordinator and the households. The negotiation will take place several times during the day, as the households may deviate from their scheduled power usage profile.

In order to simplify the notation, in the following we will consider that each household consists of just one appliance, although the proposed solution still applies to the general case. Thus, in this simplified setting, the coordinator and the households solve the following problem:

$$J = \min_{\{x_i\}_i} \left[\sum_{i=1}^{N} f_i(x_i) + g\left(\sum_{i=1}^{N} x_i \right) \right], \tag{4}$$

where in the remainder of the paper x_i represents a household with a single appliance, and $f_i(x_i)$ is the model of a single appliance.

C. Receding horizon

The problem formulated in Eq. (4) is solved with a receding horizon, using the following procedure: at each time step a new control policy is found for a given future time range, but that policy is used only in the next time step.

We write $x_i^{(t_1,t_2)} = (x_i(t_1), \dots, x_i(t_2)) \in \mathbb{R}^{t_2-t_1+1}$, $1 \leq t_1 \leq t_2 \leq T$ to refer to a slice of the power profile x_i. Then, at each time step $t = 1, \dots T-1$, a new schedule is found for the time range $[t+1, T]$ by solving the following finite horizon problem:

$$J_t = \min_{\{x_i^{(t+1,T)}\}_i} \left[\sum_{i=1}^{N} f_i^t(x_i) + g\left(\sum_{i=1}^{N} x_i \right) \right], \tag{5}$$

with $x_i^{(t+1,T)}$ the planned power usage profiles of the i-th household, and $x_i = (x_i^{(1,t)}, x_i^{(t+1,T)})$. Note that at time t we can only aim to balance the future power profiles $x_i^{(t+1,T)}$, and that $f_i^t(x_i)$ is the model of household i available at time t. The policy obtained from this minimization problem is then used at time step $t+1$, and this procedure is repeated for all $t = 0, \dots, T-1$. Note that $g(\cdot)$ could also change over time, though for simplicity we consider it is fixed.

Day-ahead schedule: At time step $t = 0$, a the day-ahead schedule [11] is found for J_0, or equivalently:

$$\{x_i^{(1,T)^*}\}_i = \arg\min_{\{x_i^{(1,T)}\}_i} \left[\sum_{i=1}^{N} f_i^0(x_i) + g^0\left(\sum_{i=1}^{N} x_i \right) \right], \tag{6}$$

with $f_i^0(x_i)$ the day-ahead model of the household i and $g^0(\sum_{i=1}^{N} x_i)$ the day-ahead global cost. The found solution can be used as an aggregated reference profile of the community for the rest of the day: $\sum_{i=1}^{N} r_i = \sum_{i=1}^{N} x_i^{(1,T)^*}$.

D. Distributed Optimization

As mentioned, at each time step t we aim to solve Eq. (5). The goal is to solve this problem in a distributed way. For this, let z_i be duplicate variables of x_i, thus:

$$\begin{aligned} J = \min_{\{x_i, z_i\}_i} & \left[\sum_{i=1}^{N} f_i(x_i) + g\left(\sum_{i=1}^{N} z_i \right) \right] \\ \text{s.t } & z_i = x_i \ \forall i, \end{aligned} \tag{7}$$

where we have omitted the time index t for clarity. Letting $\nu_i = \frac{\lambda_i}{\rho}$ be the scaled Lagrangian multipliers, we get the corresponding augmented Lagrangian in scaled form of J (refer to [25], [11] for details):

$$\begin{aligned} \tilde{J} = \min_{\{x_i, z_i\}_i} & \left[\sum_{i=1}^{N} f_i(x_i) + g\left(\sum_{i=1}^{N} z_i \right) \right. \\ & \left. + \frac{\rho}{2} \sum_{i=1}^{N} ||x_i - z_i + \nu_i||^2 - \frac{\rho}{2} \sum_{i=1}^{N} ||\nu_i||^2 \right]. \end{aligned} \tag{8}$$

Using this scaled form of the augmented Lagrangian, we now use the ADMM algorithm [25], which decomposes this optimization problem in a three step iterative procedure. In Eq. (7) there are individual costs $f_i(x_i)$ and a shared cost $g(\sum_i x_i)$, formulation that corresponds to a canonical problem known as the *sharing problem*. In this case, by applying the ADMM procedure, Eq. (8) is solved by the following three-step iterative procedure. At iteration k:

$$
\begin{aligned}
x_i^{(k+1)} &:= \arg\min_{x_i} \left[f_i(x_i) + \frac{\rho}{2} ||x_i - z_i^{(k)} + \nu_i^{(k)}||^2 \right] \\
z_i^{(k+1)} &:= \arg\min_{z} \left[g(\sum_i z_i) + \frac{\rho}{2} \sum_i ||z_i - x_i^{(k+1)} - \nu_i^{(k)}||^2 \right] \\
\nu_i^{(k+1)} &:= \nu_i^{(k)} + x_i^{(k+1)} - z_i^{(k+1)}.
\end{aligned}
\tag{9}
$$

By noting that we only need to use the average of z_i, $\bar{z}$, that $\nu_i^{(k)} = \nu^{(k)}\ \forall i$ (see [25] for details), and by defining $b^{(k)} = \bar{x}^{(k)} - \bar{z}^{(k)} + \nu^{(k)}$ (with $\bar{q} = \frac{1}{N}\sum_i q_i$), Eq. (9) can be rewritten as:

$$
\begin{aligned}
&1\!: x_i^{(k+1)} := \arg\min_{x_i} \left[f_i(x_i) + \frac{\rho}{2} ||x_i - x_i^{(k)} + b^{(k)}||^2 \right] \\
&2\!: \bar{z}^{(k+1)} := \arg\min_{\bar{z}} \left[g(N\bar{z}) + \frac{N\rho}{2} ||\bar{z} - \bar{x}^{(k+1)} - \nu^{(k)}||^2 \right] \\
&3\!: \nu^{(k+1)} := \nu^{(k)} + \bar{x}^{(k+1)} - \bar{z}^{(k+1)},
\end{aligned}
\tag{10}
$$

with $b^{(0)} = 0$, $\nu^{(0)} = 0$, and $x_i^{(0)} = \arg\min_{x_i} [f_i(x_i)]$.

E. Negotiation protocol

The receding horizon optimization problem stated in Eq. (5), and formulated as Eq. (7), is solved at each time step t following the iterative procedure in Eq. (10). In this iterative process, each household solves the step 1 associated to its own power profile, while the the coordinator solves the step two and three. This iterative process can be seen as negotiation, where at iteration k:

First, each household $i = 1, \ldots, N$

- obtains its power usage profile $x_i^{(k+1)}$ using the current broadcast signal $b^{(k)}$ (step 1, Eq. (10)).

Second, the coordinator

- collects the profiles $x_i^{(k+1)}, i = 1, \ldots, N$,
- averages the profiles $\bar{x}^{(k+1)} = \frac{1}{N}\sum_i x_i^{(k+1)}$,
- calculates $\bar{z}^{k+1}$ (step 2, Eq. (10)),
- updates the dual variables $\nu^{(k+1)}$ (step 3, Eq. (10)), and
- broadcasts $b^{(k+1)}$ to all households.

This negotiation goes on until the number of maximum iterations, K, is achieved, or until the stopping criteria is achieved (bounds in the primal and dual residuals [25]). This algorithm is synchronous at each iteration, with the coordinator waiting for the profiles of all households, and the households waiting to receive the broadcast signal from the coordination to generate new power profiles.

There are two important things to note. First, only the coordinator has access to the power usage profile of the households (households do not have access to each other profiles). This has the advantage of reducing the required communication bandwidth, and also gives more privacy to the users. In addition, the coordinator has access only to the profile of each household, but it does not have access to the individual cost functions, nor it can control the appliances. Secondly, the coordinator communicates the same broadcast signal to all households, thus reducing the communication bandwidth, but also giving symmetric information to all households.

F. Generative probabilistic model of the appliances

The control of the appliances is done by the household, which has access to the appliance model $f_i^t(x_i)$ at time t. In the following we will omit the time subscript t for clarity. We recall that the appliance model is:

$$
f_i(x_i) = \min_{u_i} \left[f_i^u(u_i) + f_i^{x|u}(x_i, u_i) \right]. \tag{11}
$$

We give a particular implementation that can be used to model different kinds of appliances by following [11]. We define a generative model $P(x_i, u_i) = P(u_i)P(x_i|u_i)$, with $P(u_i)$ a measure of how natural is control u_i, and $P(x_i|u_i)$ a measure of the certainty of achieving a given power usage profile given a control signal u_i. Therefore we define:

$$
\begin{aligned}
f_i^u(u_i) &\doteq -\log P(u_i), \\
f_i^{x|u}(x_i, u_i) &\doteq -\log P(x_i|u_i), \\
f_i(x_i) &\doteq \min_{u_i \in \mathcal{U}_i} \left[-\log P(x_i, u_i) \right].
\end{aligned}
\tag{12}
$$

In the cases when $P(x_i, u_i)$ becomes zero, $f_i(x_i)$ becomes $+\infty$, indicating that the associated profile x_i is not achievable. A particular instance of this is when $P(x_i|u_i)$ is given by the delta function $\delta(x_i - \chi_i(u_i))$, with $\chi_i : \mathcal{U}_i \to \mathbb{R}^T$.

A key aspect in the balance of the power profiles is to schedule the power usage of the households and appliances. In many cases, the usage timing is more important (and also more controllable) than the power level used. Thus, in this work we focus on the usage timing of the appliances. For this, we model the probability of the control u_i, $P(u_i)$, using a hidden semi-Markov model (HSMM) [34][35]. This method explicitly models time-varying signals as a sequence of time intervals (or modes) and is popular in the speech recognition community.

Following HSMM, we model the profile x_i by a sequence of discrete states $s_{i,t} \in \mathcal{Q}_i \doteq \{q_{i,m}\}_{m=1,\ldots,M_i}$, where each of these states, $s_{i,t}$, represents the state of appliance i at time t (so called modes). We assume that the control variables u_i are these modes at time $t = 1, \ldots, T$. For clarity, in the following we will omit the i index and we will use y_t instead of $x_{i,t}$ to avoid the abuse of symbol x. Thus $x_i = (y_1, \ldots, y_T) = y_{1:T}$. In a HSMM, the output at time t, y_t, is assumed to be determined only by s_t, thus:

$$
P(x_i|u_i) = P(y_{1:T}|s_{1:T}) = \prod_{t=1}^{T} P(y_t|s_t), \tag{13}
$$

where $P(y_t|s_t)$ is the output probability distribution.

On the other hand, $P(u_i) = P(s_{1:T})$ is modeled as a segment-based process. Following [36], let $\tau_t \geq 1$ be a random variable denoting the remaining time at the current mode s_t. Assume that the pair (s_t, τ_t) takes the value (q_m, d), i.e. the mode at time t current time is q_m and the remaining time is d, then:

- If $d > 1$, then $(s_{t+1}, \tau_{t+1}) = (q_m, d-1)$ (no mode transition occurs), otherwise
- If $d = 1$, then $(s_{t+1}, \tau_{t+1}) = (q_{m'}, d')$ (mode transition).

This process is modeled by a mode transition probability $P(\tau_t = d | s_t, \tau_{t-1} = 1)$. The parameters of a HSMM are:

- Initial mode: $\pi_m \doteq P(s_1 = q_m)$,
- Mode transition: $a_{mn} \doteq P(s_{t+1} = q_n | s_t = q_m, \tau = 1)$,
- Duration: $p_m(d) \doteq P(\tau_t = d | s_t = q_m, \tau_{t-1} = 1)$,
- Output: $b_m(y_t) \doteq P(y_t | s_t = q_m)$,

where $q_m, q_n \in \mathcal{Q}, d \geq 1$ and $y_t \in \mathbb{R}$. These parameters can be estimated using methods such as the EM-algorithm [36]. Given that our aim at this point is to show the effectiveness of the proposed distributed on-line coordination framework, in Section IV we will present simulation results where these parameters are not learned, but manually determined.

G. Optimal mode scheduling

The step 1 of the distributed optimization procedure described in Eq. (10) requires to solve a minimization problem that takes into account the described probabilistic generative model. The problem to be solved corresponds to:

$$x_i^{(k+1)} = \arg\min_{x_i} \left[f_i(x_i) + \frac{\rho}{2} ||x_i - x_i^{(k)} + b^{(k)}||^2 \right]. \quad (14)$$

If we note $F_i^{(k)}(x_i) \doteq \frac{\rho}{2} ||x_i - x_i^{(k)} + b^{(k)}||^2$ and include the generative model, this is equivalent to solve:

$$\begin{aligned} &\arg\min_{u_i} \left[-\log P(u_i) + \min_{x_i} \left(-\log P(x_i|u_i) + F_i^{(k)}(x_i) \right) \right] \\ &= -\arg\max_{u_i} \left[\log P(u_i) + \max_{x_i} \left(\log P(x_i|u_i) - F_i^{(k)}(x_i) \right) \right]. \end{aligned} \quad (15)$$

Solving this directly requires evaluating M^T sequences, which may be prohibitive for large numbers of modes, M, and time slots, T. However we can note that

$$\begin{aligned} &\max_{x_i} \left[\log P(x_i|u_i) - F_i^{(k)}(x_i) \right] \\ &= \max_{y_{1:T}} \left[\sum_{t=1}^{T} \log P(y_t|s_t) - F_{i,t}^{(k)}(y_t) \right]. \end{aligned} \quad (16)$$

with $F_{i,t}^{(k)}(y_t)$ the t-th component of $F_i^{(k)}(x_i)$. Then we can define $G_t(s_t) \doteq \max_{y_t} \left(\log P(y_t|s_t) - F_{i,t}^{(k)}(x_i) \right)$, thus

$$\max_{y_{1:T}} \left[\sum_{t=1}^{T} \log P(y_t|s_t) - F_{i,t}^{(k)}(y_t) \right] = \sum_{t=1}^{T} G_t(s_t). \quad (17)$$

Using this formulation, Eq. (15) can be solved efficiently by using dynamic programming. In addition [11]:

$$\begin{aligned} P(s_t = q_m, \tau_t = d) =& P(s_{t-1} = q_m, \tau_{t-1} = d+1) \\ +& P(\tau_{t-1} = 1, s_t = q_m) P(\tau_t = d | \tau_{t-1} = 1, s_t = q_m) \end{aligned} \quad (18)$$

and

$$P(\tau_{t-1} = 1, s_t = q_m) = \sum_{n \neq m} a_{nm} P(\tau_{t-1} = 1, s_{t-1} = q_n),$$

thus, to arrive at $(s_t, \tau_t) = (q_m, d)$ from s_{t-1} there are only M paths depending on s_{t-1}. Thus, by defining $\Phi_t(m, d)$ as:

$$\max_{s_{1:t-1}} \left[\log P(s_{1:t-1}, s_t = q_m, \tau_t = d) + \sum_{t'=1}^{t-1} G_{t'}(s_{t'}) \right] \quad (19)$$

we can write the following recursive formulation:

$$\begin{aligned} \Phi_t(m,d) = \max \big[&\Phi_{t-1}(m, d+1) + G_{t-1}(q_m), \\ &\log p_m(d) + \max_n \left(\Phi_{t-1}(n,1) + \log a_{nm} + G_{t-1}(q_n) \right) \big], \end{aligned} \quad (20)$$

with the maximization done over all possible s_{t-1}. From the definition, the initialization is given by

$$\Phi_1(m,d) = \log P(s_1 = q_m, \tau_1 = d) = \log p_m(d) + \log \pi_m. \quad (21)$$

The solution of Eq. (15) is given by $\max_{m,d}(\Phi_T(m,d) + G_T(q_m))$ and the corresponding mode scheduling is obtained by tracing back the optimal path of the recursion. Thus, this gives the control signal and the associated power profile of step 1 of the distributed scheduling in Eq. (9).

H. Appliance model update and adjustment

Similarly to [24] we consider that the available information regarding the appliances may change during the day. In our formulation, at time t, this information is encoded in the generative model $f_i^t(x_i)$. A simple example of a change in $f_i^t(x_i)$, consists in the case when the control $u_i = \hat{u}_i$ becomes fixed by the appliance, and that the control is informed to the household at time t (e.g. a rice cooker with a timer). Then, the probabilities in the generative model at time t becomes $P^t(u_i = \hat{u}_i) = 1$ and $P^t(u_i \neq \hat{u}_i) = 0$, and therefore $f_i^t(x_i)$ is minimized at $f_i^t(x_i) = 0$ for $u_i = \hat{u}_i$ (but becomes $f_i^t(x_i) = \infty$ for $u_i \neq \hat{u}_i$). In the ideal case, the schedule should be informed to the household as soon as fixed, so that the on-line negotiation can take this information into account and a new schedule can be planed to adjust in advance, instead of being rescheduled in a reactive way.

Also note that $f_i^t(x_i)$ must be adjusted using past observations of the power profile, that is, the observed values until time t, $x_i^{(1,t)}$. In particular, the duration probabilities must be adjusted over time. For example, if at time t the household i has not yet started to be used, the duration probability of the first mode (start time) will be zero for all values smaller or equal than t. Basically, the probability distribution of the duration becomes truncated for small values of d (earlier modes) and for large values of d (later modes).

IV. Simulation Results

We evaluate the effectiveness of the proposed framework in a simulated scenario (similar to [11]) designed to analyze the effect of two factors related to the on-line scheduling: *i*) how well the control handles the inflexibility of a group households, and *ii*) how well a notification in advance of the actual self-defined schedule of inflexible households, improves the coordinated schedule.

The scenario consists of a group of $N = 20$ households, each having a single appliance that corresponds to an electric vehicle that must be charged (3kWh) continuously for about 3 hrs during the day. We divided a day into $T = 144$, 10-minute slots, thus the charging takes about 18 time-slots. The community tries to minimize its deviation from a reference aggregated power usage during the day: $g(\sum_i x_i) = ||\sum_i x_i - r||^2$, with $r = \sum_i r_i$ the day-ahead aggregated reference power profile obtained following Eq. (6). Note that is the community the one that tries to follow the aggregated reference profile r and not each household trying to follow its own reference profile r_i. In the experiments, the day-ahead schedule is obtained at time $t = 0$ for $g^0(\sum_i x_i) = ||\sum_i x_i||^2$, i.e. the day-ahead schedule seeks to balance the aggregated power profile.

Each electric vehicle is represented by three modes ($M = 3$): $\mathcal{Q} = \{q_1, q_2, q_3\}$, with q_1 representing the period before charging the electric car, q_2 the charging period, and q_3 the period of after the charging (the remaining of the day). In other words, we assume that the initial mode and the transition probabilities are identical for all the household, with a deterministic transition pattern $q_1 \to q_2 \to q_3$. In addition, as we want to focus on the mode scheduling aspect, we consider an output distribution given by $P(y_t|s_t) = 1_{[y_t = \mathcal{X}(s_t)]}$, with $1_{[\circ]}$ the indicator function and $\mathcal{X}(s_t) = 0, 1000, 0[W]$ for $s_t = q_1, q_2, q_3$ respectively. We assume that each appliance can present one of the following two behaviors:

Flexible appliance: It follows the coordinated schedule and it has a fixed model through the day:

$$f_i^t(x_i) = f_i^0(x_i) \ \forall t.$$

Inflexible appliance: It does not follow the scheduled control, and it will inform the actual one at time $t_i' > 0$:

$$f_i^t(x_i) = \begin{cases} f_i^0(x_i), & \text{if } t < t_i' \\ f_i^{'}(x_i), & \text{if } t \geq t_i'. \end{cases} \tag{22}$$

where $f_i^{'}(x_i)$ is the cost function used by the appliances from time t_i'. An inflexible appliance can not be controlled: it has fixed duration times for the 3 modes (q_1, q_2, q_3), with the second mode (charging) starting at time $\tilde{t}_i$.

In the experiments a percentage ($p = 15\%, 30\%, 45\%$) of households are flexible (out of the N), and the coordinator does not know in advance neither p nor which households are inflexible. An inflexible appliance informs its self-defined schedule at time $t_i' = \tilde{t}_i - \delta_i$, where $\tilde{t}_i$ is the actual starting time (duration of mode 1), and where $\delta_i \in \mathbb{R}$ indicates how early in advance the self-defined scheduled will be informed to the household (relative to the usage time $\tilde{t}_i$). Thus, the larger the value of δ_i, the longer the time the community can to adjust, in advance, to unplanned schedule changes. A $\delta_i \leq 0$ means the appliance does not inform anything to the household: it just starts charging the vehicle at time $\tilde{t}_i$ and the household does not know it in advance. For the simulations we consider that δ_i follows a Gaussian distribution with mean $m_\delta \in \{0, 10, 20, 30\}$ and standard deviation equal $\sigma_\delta = 18$, while $\tilde{t}_i$ is sampled from the same distribution as the duration of mode q_1 of the appliance (defined below).

In the day-ahead model, the duration of mode m follows a Gaussian distribution $\mathcal{N}(\mu_m, \sigma_m^2)$, having the following parameters (μ_1 and μ_3 are different for every user):

- μ_1 is uniformly distributed in $[50, 55]$; $\sigma_1 = 10$.
- $\mu_2 = 18$ (3 hrs); $\sigma_2 = 1$.
- $\mu_3 = T - (\mu_1 + \mu_2)$; $\sigma_3 = 10$.

The simulation was ran 10 times, each time using different samples from the corresponding distributions. The obtained results are summarized in Figure 3, while Figure 2 presents results for a particular run for four methods:

- Without coordination (dashed black)
- Reference profile $r = \sum_i r_i$ (dot-dashed green)
- Without on-line coordination (filled blue)
- With on-line coordination (solid red) (proposed)

The results of with (red) / without (blue) on-line coordination are analyzed (Figure 3) using three measures (smaller values are better), where x_i represents the case with (without) on-line coordination:

Power balance: ratio of the ℓ_2-norm of the aggregated power without coordination ($\tilde{x}_i$) and the ℓ_2-norm of the aggregated power: $||\sum_i \tilde{x}_i||^2 \Big/ ||\sum_i x_i||^2$.

Peak Shaving: ratio of the maximum of the aggregated power power without coordination ($\tilde{x}_i$) and the maximum of the aggregated power: $\max_t(\sum_i \tilde{x}_i(t)) \Big/ \max_t(\sum_i x_i(t))$.

Deviation from reference: ℓ_2-norm of the difference of the aggregated power and the reference profile, normalized by the ℓ_2-norm of the reference: $||\sum_i x_i - r||^2 \Big/ ||r||^2$.

From the results it can be observed that: *i*) The day-ahead control policy can balance the power, reducing both the ℓ_2-norm and the maximum peak, even when only $p = 15\%$ households are flexible (13.7% ℓ_2-norm reduction and 9% peak reduction). *ii*) The on-line control can further improve the power balance, further reducing the ℓ_2-norm and the maximum peak. When δ is small (e.g. $\delta = 0$, Figure 2, first column) the control is basically reactive, postponing the schedule and thus producing a small peak rebound, but nevertheless flattening power usage. When δ is large (e.g. $\delta = 30$, Figure 2, fourth column) the control is not reactive, but re-schedules the charging of more electric vehicles before the main peak, producing no peak rebounds (achieving a 17.2% ℓ_2-norm reduction, a 15.5% peak reduction). *iii*) When $p = 45\%$ (Figure 3, third column) and the inflexible households notify their schedule well in advance ($\delta = 30$), the reduction in ℓ_2-norm and maximum peak (first and second row) are rather large (35% and 45% respectively), also achieving an aggregated profile (third row) that is much closer to the reference one (45% normalized difference from reference).

V. Conclusion

In the current paper we propose a framework for distributed on-line power demand balancing of households communities. The framework is designed such that a community of household coordinates and updates the schedule of their appliances power usage, instead of the appliances being controlled by the utility company. The goal is to balance the aggregated power of the community of households taking into account the Quality of Life (QoL) of the users. This

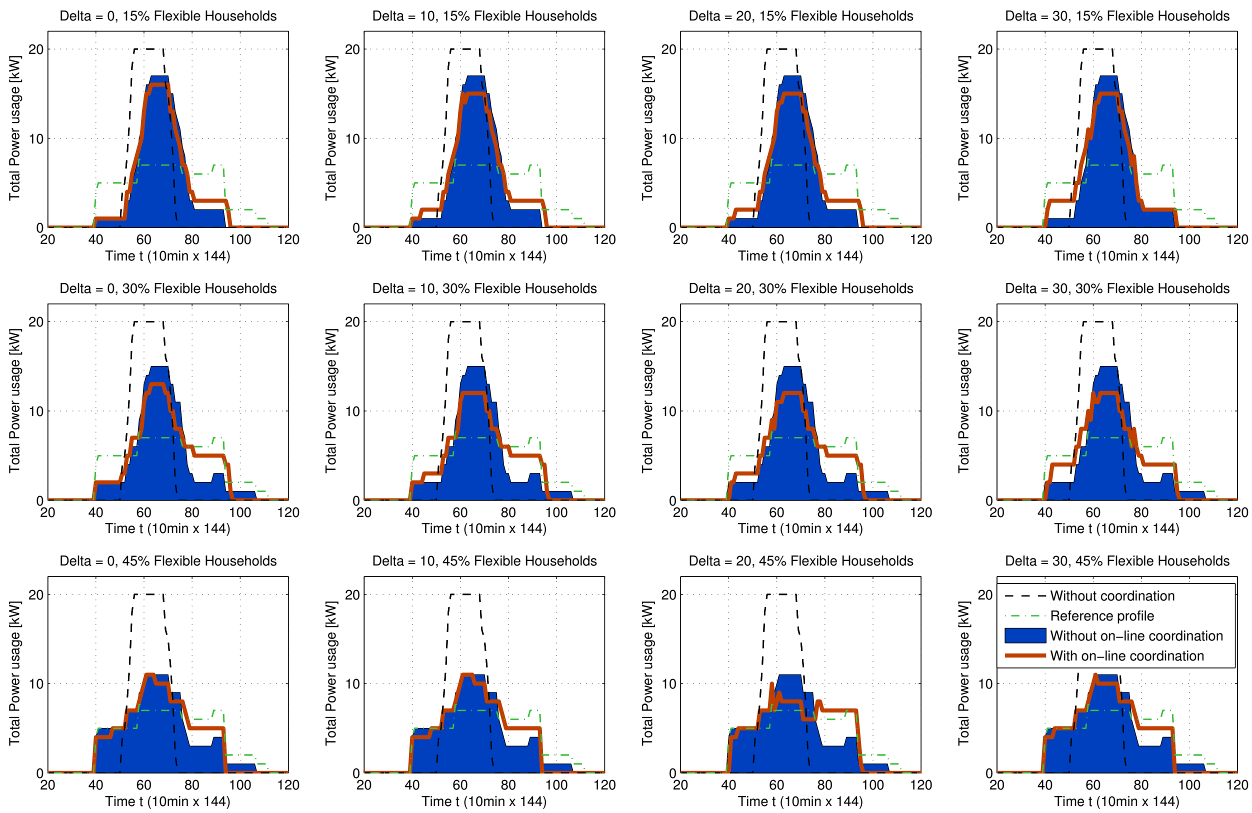

Fig. 2. Aggregated power usage of 20 electric vehicles during one day. Results for (rows) different number of flexible households ($p = 15$, 30 and 45%), and (columns) different distributions of notification time (Gaussian parametrized by mean $\delta \in \{0, 10, 20, 30\}$ and $\sigma_\delta = 18$) of inflexible households.

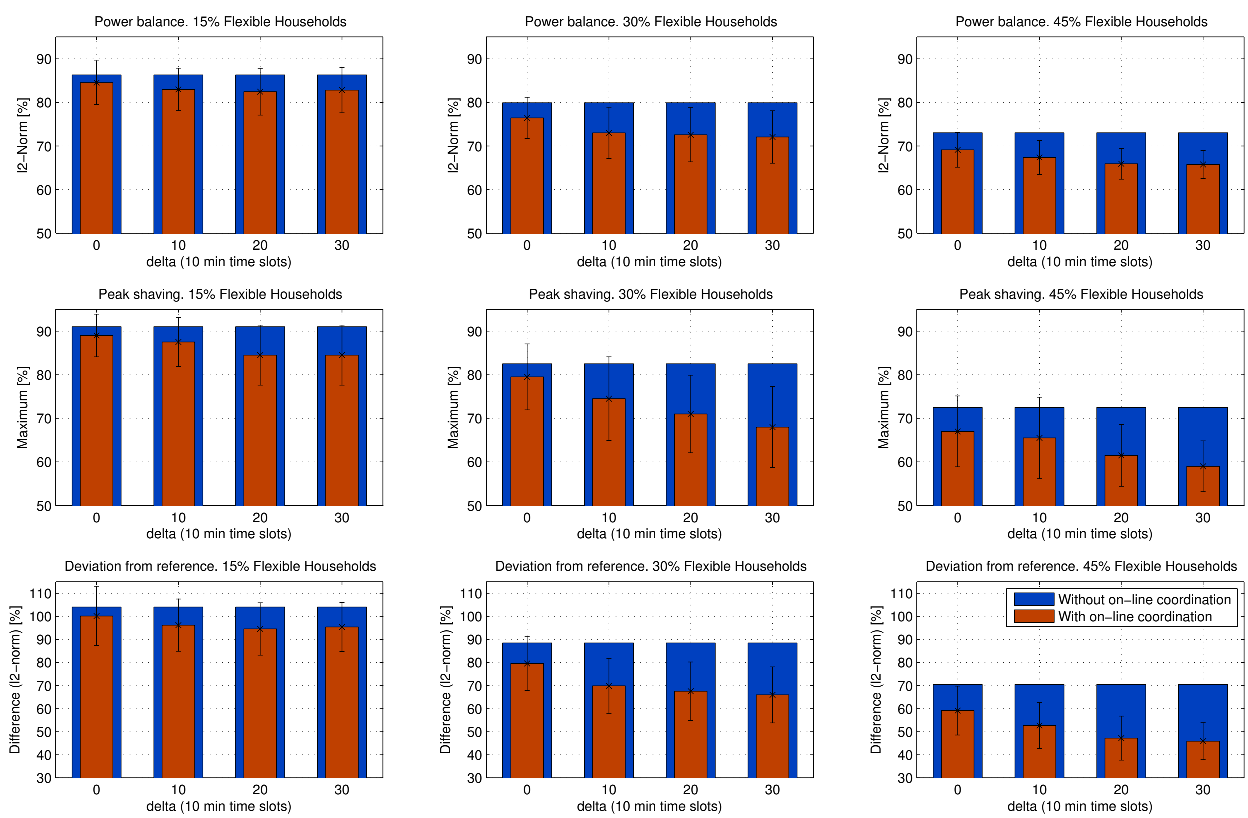

Fig. 3. Average results over 10 simulation runs. Results are indicated as percentages relative to the case when no control is applied (power balance, first row; peak shaving, second row) and to the reference profile (deviation from reference, third row). Lower values are better. Results for (columns) different number of flexible households ($p = 15$, 30 and 45%). The x-axis indicates the distribution of the notification time of inflexible households (Gaussian parametrized by mean $\delta \in \{0, 10, 20, 30\}$ and $\sigma_\delta = 18$).

is formulated as a receding horizon optimization problem where we seek not to deviate much from a reference aggregated usage profile and to minimize deviations from common usage patterns. For this, a generative probabilistic model of appliance power usage is considered to determine the optimal coordinated schedule of the appliances. We analyze the case where the households are not enforced to follow the coordinated schedule (when some of them do not, they will negotiate a new schedule for the next time step), and where the appliances may notify in advance their self-defined schedule, allowing to update the generative model with actual usage information, and thus improve the power balancing and reduce peak rebounds. As future work, and similarly to the analysis done in [37] for the decentralized consensus problem, we plan to analyze the proposed framework when the objective functions change over time.

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